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Calculation of Sheet Steel Diaphragms in the UK

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CALCULATION OF SHEET STEEL DIAPHRAGMS IN THE U.K.

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1. North American Approach

In the U.S. much of the early work on light gauge steel diaphragms was carried out by Nilson[1] at Cornell University and was supplemented by manufacturers' testing programmes. This work resulted in a very practical and specific design guide[2] published by A.I.S.I. in 1967.

In this guide, criteria are given for the design of diaphragms based on the allowable shear strength and permissible shear deflection. The allowable shear strength was obtained from ultimate load tests on panels of standard corrugated sheeting fixed with fasteners at particular spacings. The results are therefore strictly applicable only to one particular panel, though they may be used conservatively for any other weaker panels.

With regard to in-plane shear deflection, the diaphragm and edge members are considered as a plate girder. The total shear deflection is then given by

$$\Delta_{total} = \Delta_b + \Delta_s \qquad \ldots. \quad (1.1)$$

where Δ_b is the bending deflection of the plate girder

and Δ_s is the shear deflection of the web (i.e. the diaphragm)

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The value of Δ_b may be calculated by simple bending theory, but Δ_s involves the use of an effective shear modulus obtained from tests and so the method is again strictly applicable only to one configuration of sheeting and fasteners.

In Canada the foregoing approach has been extended in a publication by the Sheet Steel Building Institute[(3)]. Design tables are given for strength and deflection of $1^1/2$ inch deep steel decks of various thicknesses, spans and fastener spacings. Again, these tables are extrapolated from test results on actual decks, and although the specimen design examples are attractively simple, they are subject to the same restrictions noted earlier, in that they are not generally applicable to any other panel or fastener configuration.

It should be noted that the limitations mentioned in the design information currently available in the U.S. and Canada have been largely overcome by the finite element techniques recently used by Nilson[(4)]. However, this work has not yet reached the stage where it may be easily used by designers. At present, therefore, if it is wished to depart from a very limited range of diaphragm types, it is necessary to carry out full scale tests. This is expensive and is often not considered warranted unless the structure is very large or is the prototype for a large number of buildings. The requirement to test is therefore inhibiting and should be replaced if possible by a more generalized design method for diaphragms with, if necessary, occasional confirmatory tests to compare the calculated and observed behaviour.

2. British Approach

In Britain, work on diaphragm action has been progressing quite independently over the past fifteen years. At first it was centred at the University of Manchester but is now based at the University of Salford where it is supported by Constrado (Constructional Steel Research and Development Organisation; part of the British Steel Corporation). The work has now reached the stage where it has been incorporated in Codes of Practice and used extensively in the design of buildings. The design method is usually referred to as "Stressed Skin Design".

The approach has been to study the mode of deformation and failure of panels of corrugated steel sheeting and to derive appropriate mathematical expressions. Some of these expressions contain data which can only be obtained by tests on components (e.g. the slip value and ultimate tearing value of sheet fasteners) but they are all universally applicable. The work has been written up in a textbook[5] and disseminated through a number of Courses organised by Constrado.

As in North America, the two criteria of deflection and strength of diaphragms have been considered, and the concept of the plate girder (Fig 1) has been used. Perhaps the main difference is the emphasis which is given in Britain to the individual panel (e.g. ABCD in Fig 1). Thus, in considering diaphragm deflection, each panel may be represented by an equivalent spring (Fig 2) which includes both the bending and shear deflection items contained in equation (1.1). For deflection calculations the "shear flexibility" of a panel is used rather than the "shear stiffness". This concept is illustrated in Fig 3 which shows that the shear flexibility c of a panel is the shear deflection per unit load in the direction of the corrugations.

The convenience of the term shear flexibility may be appreciated by referring to the analogy of the equivalent spring shown in Fig 4. In a practical shear panel there are a number of component flexibilities due to sheet deformation (c_1), slip in the sheet fasteners (c_2), and axial strain in the edge members i.e. the "bending" flexibility (c_3). These component flexibilities may then be directly added together to give the total shear flexibility c. In practice, these main flexibilities are further sub divided as shown in Fig 4, but the principle of superposition is maintained.

With regard to diaphragm strength, the criterion must be the least strength of the diaphragm or its edge members. Thus, considering the diaphragm as a plate girder (Fig 1) the panel ABCD will be critical with regard to shear and so ideally the sheet should be checked for shear strength, local and overall shear buckling, and for failure at the seam or sidelap fasteners and at the sheet/steelwork fasteners. In Britain, as in most European countries, welding of the deck to the supporting steelwork is rarely used, and attachment is by means of cartridge fired pins or self tapping screws. Under these circumstances, failure of the end panel in shear is almost invariably due to tearing at the fasteners, and simple design expressions based on this assumption have been formulated.

In addition to the shear strength of the diaphragm, the axial strength of the edge members and their joints particularly at the centre of length of the diaphragm should be checked. Buckling of the edge member under compression should also be considered, but the lateral support provided by the decking is often of considerable assistance in this connection.

3. Shear Flexibility of a Panel

Consider the typical shear panel shown in Fig 5 in which the various components are defined. It will be noted that this panel is fastened on all four edges. This method of attachment is strongly advocated wherever possible, as it leads to much stronger and stiffer diaphragms. The information given in this paper refers only to panels fastened on four edges. A treatment of panels fastened on two edges only (i.e. with the "shear-connectors" omitted) was originally given in reference 5, but this has been superseded by a more exact method put forward by Davies[6].

For panels which are fastened on all four edges, the component flexibilities referred to in Fig 4 may be calculated by means of the expressions given in Tables 1 and 2. These expressions have been derived in reference 5 and the notation is explained in Table 3 with reference to an actual example. The cases considered in Tables 1 and 2 are as follows:

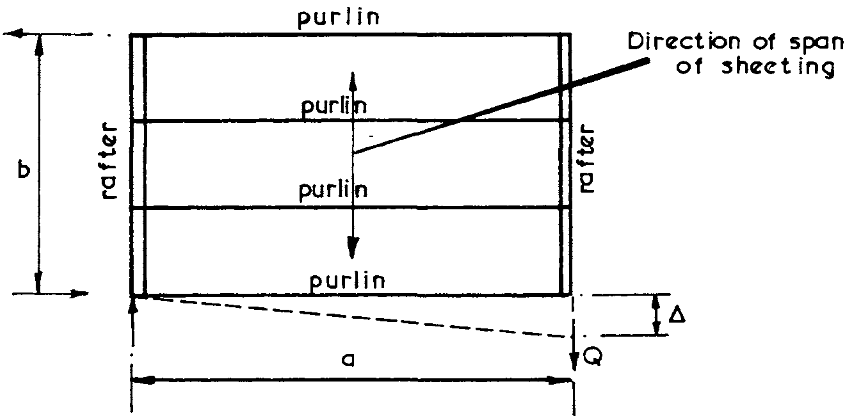
Table 1. Individual panel: sheeting spanning between purlins

Table 2. Multi-panel diaphragm: sheeting spanning between purlins

Explanatory notes on the component shear flexibilities are now given, together with notes on the use of Tables 1 and 2.

3.1 Sheet Distortion $c_{1.1}$ Shear flow round the profile produces distortion of the corrugation as shown in Fig 6a when the corrugation is fastened in every trough or as shown in Fig 6b when fastened in alternate troughs. The expression for shear flexibility depends on the sheeting constant K which in turn depends very much on the frequency of fixing. Calculated values of K for a wide range of profiles and various fastener spacings are given in reference 5.

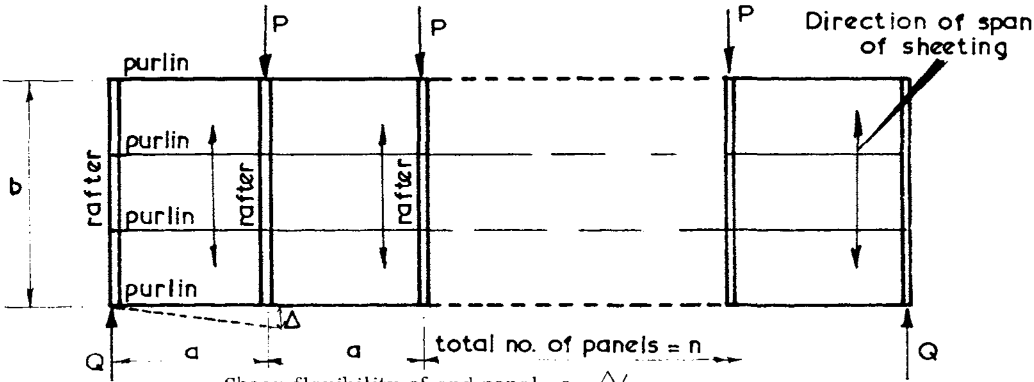
3.2 Shear Strain $c_{1.2}$ The surface of the sheeting is comprised of a series of long thin rectangles on each face of the profile. Under shear strain each of these rectangles tends to distort into a parallelogram. The expression given takes care of this effect; strictly it is correct only for rectangular corrugations but it gives



Shear flexibility $c = \Delta/Q$

Shear flexibility due to		Sheeting fixed to purlins and shear connectors: Expression	Flexibility mm/kN
(1) Sheet deformation	Sheet distortion	$c_{1.1} = \dfrac{0.144ad^4 f_1 K}{Et^3 b^3}$	$c_{1.1} =$
	Shear strain	$c_{1.2} = \dfrac{2af_2(1+\nu)(1+2h/d)}{btE}$	$c_{1.2} =$
(2) Sheet fasteners	Sheet/purlin fasteners	$c_{2.1} = \dfrac{2aspf_3}{b^2}$	$c_{2.1} =$
	Seam fasteners	$c_{2.2} = \dfrac{(n_{sh} - 1)s_s}{n_s}$	$c_{2.2} =$
	Sheet/shear connector fasteners	$c_{2.3} = \dfrac{2s_{sc}}{n_{sc}}$	$c_{2.3} =$
(3) Flange forces	Axial strain in purlins	$c_3 = \dfrac{2a^3 f_3}{3b^2 AE}$	$c_3 =$
		Total shear flexibility	$c =$

Table 1. Shear flexibility of isolated panel with sheeting spanning between purlins



Shear flexibility of end panel, $c = \Delta/Q$

Shear flexibility due to		Sheeting fixed to purlins and shear connectors	
		Expression	Flexibility mm/kN
(1) Sheet deformation	Sheet distortion	$c_{1.1} = \dfrac{0.144ad^4 f_1 K}{Et^3 b^3}$	$c_{1.1} =$
	Shear strain	$c_{1.2} = \dfrac{2af_2(1+\nu)(1+{}^{2h}/d)}{btE}$	$c_{1.2} =$
(2) Sheet fasteners	Sheet/purlin fasteners	$c_{2.1} = \dfrac{2aspf_3}{b^2}$	$c_{2.1} =$
	Seam fasteners	$c_{2.2} = \dfrac{(n_{sh} - 1)s_s}{n_s}$	$c_{2.2} =$
	Sheet/shear connector fasteners	$c_{2.3} = \dfrac{2s_{sc}}{n_{sc}}$	$c_{2.3} =$
(3) Flange forces	Axial strain in purlins	$c_3 = \dfrac{n^2 a^3 f_3}{6EAb^2}$	$c_3 =$
		Total shear flexibility	$c =$

Table 2. Shear flexibility of continuous panel with sheeting spanning between purlins

conservative results for trapezoidal profiles.

3.3 Sheet/Purlin Fasteners $c_{2.1}$ This effect is illustrated in Fig 7. The shear flexibility from this cause depends on the pitch and the slip of the fasteners. Extensive tests on a range of fasteners were carried out to obtain the recommended slip values for use in design, given in reference 5.

3.4 Seam (Sidelap) Fasteners $c_{2.2}$ Slip in these fasteners produces the effect shown in Fig 8. If all the fasteners are similar, the expressions given in Tables 1 and 2 are correct, but in practice seams are often secured not only by the sidelap fasteners but also by the sheet/purlin fasteners which pass through both sheet thicknesses at the overlap. If the slip characteristics of these two types of fastener are different, the equivalent number of seam fasteners must be found.

3.5 Sheet/Shear Connector Fasteners $c_{2.3}$ If the tops of the rafters are not level with the tops of the purlins it will be necessary to use shear connectors over the rafters (as in Fig 5) to ensure that the panel is fastened on all four sides. Slip in the sheet/shear connector fasteners is illustrated in Fig 9.

3.6 Intermediate Purlins: Factors f_1, f_2 and f_3 In the expressions for component shear flexibility, the factors f_1, f_2 and f_3 allow for the effect of intermediate purlins in a panel. Values are given in reference 5. Where intermediate purlins occur, the shear across a panel is not uniform but increases towards the mid depth. Moreover axial forces are set up in the intermediate purlins provided an equal and opposite reaction can be mobilised. In the diagram in Table 1 it is seen that any such axial forces in the purlins would cause bending of the rafters about their minor axes and so it is doubtful if this effect exists in an individual panel. It can, of course, exist in multiple panel diaphragms, as in Table 2.

3.7 Axial Strain in Purlins c_3 This component is unlike the previous ones considered in that it refers to bending rather than shear. Nevertheless it can be expressed in shear flexibility terms by calculating the difference in bending deflection over a panel length (in multi-panel diaphragms), and dividing by the shear in the panel. This equivalent shear flexibility varies along the length of the building and is in fact greatest at the centre of length. Since, however, the end panel is the usual criterion, the equivalent shear flexibility is considered for this case.

The derivation of the equivalent shear flexibility for bending of a beam is given in Appendix 2. The results are compared with the more specific results for an equivalent truss and shown to be in reasonable agreement.

This procedure of considering the bending flexibility as an equivalent shear flexibility is therefore quite adequate for most practical purposes and is certainly a computational convenience.

3.8 Table 1 This is the basic case for the calculation of the shear flexibility of a panel. It is again emphasized that the expressions give the shear flexibility in a direction parallel to the corrugations.

3.9 Table 2 This case of a multi-panel diaphragm will occur much more frequently in practice. The expression for $c_{2.3}$ implies that the flexibility due to slip at the sheet/shear connector fasteners is the same at each rafter, although the load transmitted through the end rafter is Q and the load transmitted through each intermediate rafter is only P (see the diagram in Table 2). Hence a reduced number of sheet/shear connector fasteners may strictly be used on intermediate rafters as follows:

$$\frac{\text{Number of fasteners on end rafter}}{\text{Number of fasteners on intermediate rafter}} = \frac{Q}{P} = \frac{n}{2}$$

In practice, more fasteners are often used on intermediate rafters than specified above, and so the criterion is then the end panel. All other panels are on the safe side and so this is an additional reason for applying c_3 (paragraph 3.7) only to the end panel.

4. Shear Strength of a Panel

It has been mentioned in Section 2 that the most probable mode of failure of the type of shear panel used in Britain in by tearing at the sheet fasteners. Nevertheless, the other possible modes of failure i.e. local and overall shear buckling, joint failure and buckling of the edge members, should also be checked. In this Section only tearing at the fasteners will be considered. As in Section 3, the panel is assumed to be fastened on all four edges.

Reference is made to the typical shear panel shown in Fig 10. The shear strength of the panel is calculated considering failure in

(4.1) the sheet/purlin fasteners

(4.2) the seam fasteners and

(4.3) the sheet/shear connector fasteners

and the lowest value is taken as the ultimate shear strength of the panel.

In the calculations it is assumed that failure at each individual fastener is ductile, i.e. failure occurs by tearing of the sheeting rather than by shear of the fastener, so that at failure each fastener carries the same load.

The nomenclature used is as follows:-

b = depth of shear panel

f_3 = reduction factor for intermediate purlins (see reference 5)

F_p = tearing strength of sheet/purlin fastener

F_s = tearing strength of seam fastener

F_{sc} = tearing strength of sheet/shear connector fastener

n_p = number of purlins

n_s = number of seam fasteners per seam

p = pitch of sheet/purlin fasteners

Q = shear strength of panel

4.1 Failure in the Sheet/Purlin Fasteners When there are no intermediate purlins, the shear force per fastener is $\frac{Qp}{b}$ so that at failure this equals F_p,

i.e. $$Q = \frac{F_p b}{p} \qquad \text{.... (4.1)}$$

When intermediate purlins are present, this is modified to

$$Q = \frac{F_p b}{p f_3} \qquad \text{.... (4.2)}$$

4.2 Failure in the Seam Fasteners If the seam fasteners are all similar, then

$$Q = F_s n_s \qquad \text{.... (4.3)}$$

but if the sheet/purlin fasteners pass through both sheet thicknesses at the overlap, then

$$Q = F_s n_s + F_p n_p \qquad \text{.... (4.4)}$$

4.3 Failure in the Sheet/Shear Connector Fasteners

$$Q = F_{sc} n_{sc} \qquad \text{.... (4.5)}$$

Thus the ultimate shear strength, Q_{ult}, of the panel is the lowest value of Q given by equations (4.1) to (4.5).

5. Correlation with Test Results

The foregoing method for calculating the shear flexibility and shear strength of a diaphragm is now illustrated by means of an example and the results are compared with the observed values taken during a demonstration test at one of the Stressed Skin Courses organised by Constrado (Fig 11).

5.1 Data

The arrangement of the shear panel is shown in Fig 12. The remaining data are as follows:

Sheeting:	4 sheets of 38mm deep steel decking, corrugation pitch 152mm, net steel thickness 0.68mm
Purlins:	4 x 140mm deep x 2mm thick zed purlins, cross sectional area 494 mm^2
Fasteners:	Sheet/purlin fasteners - 6.1mm dia. self tapping screws. Two cases were tested:- Case (a) Screws in every corrugation Case (b) Screws in alternate corrugations Seam fasteners - 4.1mm dia. self-drilling/tapping screws. Sheet/shear connector fasteners - 6.1mm dia. self tapping screws.

Elastic Constants: For steel take the modulus of elasticity $E = 207$ kN/mm^2 and Poisson's ratio $\nu = 0.25$

Other Constants: The factors f_1, f_2 and f_3, and the sheeting constant K are determined by calculation and are listed in reference 5.

Fastener Data: The slip values and ultimate tearing strengths of fasteners have been obtained from tests and are given in reference 5. The relevant values provisionally suggested for design purposes are as follows:-

Type of fastener	Slip value (mm/kN)	Ultimate strength in kN per mm sheet thickness
6.1mm self tapping screw	0.35	6
4.1mm self drilling/tapping screw	0.35	2.5

5.2 Calculation of Shear Flexibility Using the values given in Table 3, the expressions given in Table 1 were used to calculate the shear flexibility as shown in Table 4.

5.3 Calculation of Shear Strength For fasteners in 0.68mm thick sheet, the tearing strengths are as follows:-

$$F_p = F_{sc} = 6 \times 0.68 = 4.08 \text{ kN}$$

and $$F_s = 2.5 \times 0.68 = 1.70 \text{ kN}$$

The shear strengths in the various failure modes may then be calculated. For Case (b), fasteners in alternate corrugations, these are as follows:-

Symbol	Meaning		Value Case (a)	Value Case (b)
a	length of shear panel	mm	3670	3670
A	area of purlin	mm^2	494	494
b	depth of shear panel	mm	3780	3780
d	pitch of corrugations	mm	152	152
f_1	) factors to allow for the effect of		0.97	0.97
f_2	) intermediate purlins		0.75	0.75
f_3	) (see reference 5)		0.90	0.90
h	height of corrugation	mm	38	38
K	sheeting constant (see reference 5)		0.23	9.10
n_p	number of purlins		4	4
n_s	number of seam fasteners per seam		9+4	9+4
n_{sc}	number of sheet/shear connector fasteners		9	9
n_{sh}	number of sheet widths per panel		4	4
p	pitch of sheet/purlin fasteners	mm	152	304
s	slip per sheet/purlin fastener	mm/kN	0.35	0.35
s_s	slip per seam fastener	mm/kN	0.35	0.35
s_{sc}	slip per sheet/shear connector fastener	mm/kN	0.35	0.35
t	net sheet steel thickness	mm	0.68	0.68

Table 3 Definition of symbols used in Tables 1 and 2 and values for the example in Section 5

Shear flexibility due to		Value of shear flexibility mm/kN	
		Case (a)	Case (b)
Sheet distortion	$c_{1.1}$	0.018	0.706
Shear strain	$c_{1.2}$	0.019	0.019
Sheet/purlin fasteners	$c_{2.1}$	0.025	0.049
Seam fasteners	$c_{2.2}$	0.081	0.081
Sheet/shear connector fasteners	$c_{2.3}$	0.078	0.078
Axial strain in purlins	c_3	0.020	0.020
Calculated total shear flexibility c mm/kN		0.241	0.953

Table 4 Calculated shear flexibility of panel

Sheet/purlin fasteners:

From equation (4.2), $$Q = \frac{4.08 \times 3780}{304 \times 0.90} = 56 \text{ kN}$$

Seam fasteners:

From equation (4.4), $$Q = 1.7 \times 9 + 4.08 \times 4 = 32 \text{ kN}$$

Sheet/shear connector fasteners:

From equation (4.5), $$Q = 4.08 \times 9 = 37 \text{ kN}$$

The calculated ultimate shear strength is therefore the least of these, i.e. $Q_{ult} = 32$ kN.

5.4 Observed Shear Flexibility The complete shear load/shear deflection curve for the panel, with fasteners in alternate corrugations, is shown in Fig 13. Prior to this, elastic loading tests (up to about half the ultimate load) were carried out for panels with fasteners in every corrugation and alternate corrugations. The results are in satisfactory agreement with the calculated values:-

	Shear flexibility mm/kN	
	Calculated	Observed
Case (a) - every corrugation	0.24	0.26
Case (b) - alternate corrugations	0.95	0.93

5.5 Observed Shear Strength The panel failed at a load of 34 kN which was in satisfactory agreement with the calculated value of 32 kN. The mode of failure was by tearing at the seam fasteners (Fig 14) which was also in agreement with the calculations. It is seen from Fig 13 that the failure mode is adequately ductile.

6. Application to Practical Roof Decks

The foregoing principles may be applied without difficulty to roof decks in practice. An illustrative example is given in Appendix 3 of a roof deck which spans directly between the rafters of a framed building. The correction necessary to allow for this changed direction of span is indicated.

In the example, two cases are considered:-

(1) in which the supporting frames (except the gables) are pinjointed so that the decking carries all the horizontal load

(2) in which the supporting frames are rigid so that the horizontal load is shared between the decking and the frames.

In each case, the maximum deflection of the roof deck and the load factor against failure is found.

7. Current Position

The methods described have been in use for several years and they perhaps over-simplify what is a very complex problem. Further corrections are at present being investigated by Davies[(6)] and Lawson[(7)] at Salford. Although it is not yet appropriate to discuss these in detail, an outline will be given of the effects being considered.

7.1 Effect of Separate Sheets In the example considered in Section 5, the sheet length was the same as the depth of the panel. If there are n sheet lengths in the depth of a panel, it has been shown by Lawson[(7)] that a multiplying factor of the form $(1 + \alpha n^2)$ should be applied to $c_{1.1}$, the shear flexibility due to sheet distortion.

The value of α usually lies between 0.1 and 0.3, so that a value of 0.2 will be assumed for the present purpose.

If there are three sheet lengths in the depth of the panel, the multiplying factor for $c_{1.1}$ becomes 2.8, and the resulting calculated shear flexibilities are obtained by modifying the appropriate values in Table 4. These results are compared below with the observed shear flexibilities from a test on the same panel as previously but with the sheeting replaced by three separate sheet lengths (Fig 15).

	Shear flexibility mm/kN	
	Calculated	Observed
Case (a) - every corrugation	0.27	0.28
Case (b) - alternate corrugations	2.22	1.56

In the above results, better agreement would have been obtained if a value of $\alpha = 0.1$ had been used, but at least the exercise serves to demonstrate the principles involved. Failure of the test panel occurred in a composite mode at a load of 28 kN, slightly below the previous ultimate load of 34 kN.

7.2 Effect of Sheet Length In the initial calculation of the sheeting constant K, it was assumed that the amount of distortion of the corrugation was proportional to the distance from the centre of length of the corrugation. In fact, Davies and Lawson have shown that this is not so[(8)] and that more distortion occurs within a short distance of the free ends. Thus K will depend on the actual length of the corrugation.

A mitigating factor in the increased K values resulting from the above may well be the restraint imposed by the purlins to the free distortion of the corrugations. All these effects are at present being studied.

7.3 Effect of Insulation Insulation board bonded to metal roof decks with hot bitumen can have a remarkable effect in preventing the free shear distortion of the decking, and may consequently reduce this component of the shear flexibility very substantially. This effect has been studied by Lapin(9) but firm recommendations on when it may be used have not yet been made. In any case, it is not suggested that the insultation should be relied upon to improve the shear strength.

7.4 Effect of Roof Light Openings Although some guidance has been given(10) on the position and percentage area of roof light openings which can be permitted in a roof deck diaphragm, the information is based on a limited number of tests and cannot be regarded as authoritative. Finite element analysis, as proposed by Davies, should yield information which is more conclusive, but such results will not be available for some time.

7.5 Other Effects Other practical features of roof decks may also have important effects on the shear flexibility and shear strength. For instance, decking is often fastened in every corrugation at the edges, but only in alternate corrugations at the intermediate purlins. The effect this may have on the sheeting constant K is not yet clear.

Another practical consideration is that when decking spans between the rafters, it is often continuous over two or more spans. Simplified expressions for this effect must be obtained.

From the above list, it is clear that the theoretical simulation of a practical roof diaphragm is indeed a complicated problem. Nevertheless, it is hoped to retain the original simple concepts set out in Section 2, and make the necessary corrections by means of suitable correction factors. By this means the designer will still be able to have a feeling for the behaviour of the structure and will be able to exercise his engineering judgement as to which effects are important and which are not.

8. Conclusions

The paper shows how a generalised method for the calculation of the shear flexibility and shear strength of sheet steel diaphragms has been developed. The principles of the method are attractively simple and it is shown how the concept can be used to represent the behaviour of even a complex practical diaphragm by applying certain correction factors. The method is confirmed by test results and illustrated by a design example.

Although the method has reached the stage where it has been used in design for several years, it is still being refined and should be even more powerful in the future.

Summary

The development of a simple generalised method for calculating the shear deflection and shear strength of practical diaphragms is described. The calculated behaviour is compared with test results and a practical design example is given. The method is already widely used but planned refinements should make it even more powerful.

REFERENCES

1. Nilson, A H. "Shear diaphragms of light gage steel" Journal of the Structural Division, ASCE, Vol 86, No ST 11, Nov 1960 pp 111-139.

2. American Iron and Steel Institute, "Design of light gage steel diaphragms", AISI, N.Y. 1967.

3. Canadian Sheet Steel Building Institute, "Diaphragm action of cellular steel floor and roof deck construction", CSSBI, Mississauga, Ontario, Dec 1972.

4. Nilson, A H. "Analysis of light gage steel shear diaphragms", Second Speciality Conference on Cold-Formed Steel Structures, University of Missouri-Rolla, Oct 1973 pp 325-363.

5. Bryan, E R. "The stressed skin design of steel buildings", Crosby Lockwood Staples, London 1973.

6. Davies, J M. "The design of shear diaphragms of corrugated steel sheeting", University of Salford, Department of Civil Engineering Report No 74/50, Sept 1974.

7. Lawson, R M. "Some practical design factors for the shear flexibility of corrugated sheeting", University of Salford, Department of Civil Engineering Report No 74/51, Sept 1974.

8. Davies, J M and Lawson R M. "The shear flexibility of corrugated steel sheeting", Third Speciality Conference on Cold-Formed Steel Structures, University of Missouri-Rolla, Nov 1975.

9. Lapin D. "Behaviour of insulated metal decking under shear load", MSc thesis, University of Salford, 1974.

10. Bryan E R. "Stressed skin roof decks for SEAC and CLASP building systems". Constrado publication, London 1973.

APPENDIX I - NOTATION

a = length of shear panel (mm)

A = cross sectional area of edge member (mm^2)

b = depth of shear panel (mm)

c = shear flexibility of panel (mm/kN)

(with various subscripts to indicate component shear flexibilities)

d = pitch of corrugations (mm)

E = modulus of elasticity (kN/mm^2)

f_1, f_2, f_3 = factors to allow for the effect of intermediate purlins

F_p = tearing strength of sheet/purlin fastener (kN)

F_s = tearing strength of seam fastener (kN)

F_{sc} = tearing strength of sheet/shear connector fastener (kN)

h = height of corrugation (mm)

K = sheeting constant

n number of panels in a diaphragm or

number of sheet lengths in the depth of a panel

n_p = number of purlins

n_s = number of seam fasteners per seam

n_{sc} = number of sheet/shear connector fasteners

n_{sh} = number of sheet widths per panel

p = pitch of sheet/purlin fasteners (mm)

P = load at a panel point in a multi-panel diaphragm (kN)

Q = shear strength of panel or shear force (kN)

s	=	slip per sheet/purlin fastener (mm/kN)
s_s	=	slip per seam fastener (mm/kN)
s_{sc}	=	slip per sheet/shear connector fastener (mm/kN)
t	=	net thickness of steel sheet excluding coatings (mm)
α	=	a constant
Δ	=	shear deflection of panel (mm)
ν	=	Poisson's ratio

APPENDIX 2 - EQUIVALENT SHEAR FLEXIBILITY FOR BENDING (AXIAL STRAIN IN PURLINS)

Approximate Beam Theory

Consider the simply supported beam under uniformly distributed load w per unit length, shown in Fig 16. The deflection y at any distance x is given by

$$y = \frac{w}{24EI}\left(x^4 - 2Lx^3 + L^3x\right) \qquad \ldots . \quad (A2.1)$$

In an element of δx, the increment of deflection is given by δy where

$$\frac{\delta y}{\delta x} = \frac{w}{24EI}\left(4x^3 - 6Lx^2 + L^3\right) \qquad \ldots . \quad (A2.2)$$

and the shear at distance x is $\frac{wL}{2} - wx$.

Hence the shear flexibility in this element due to bending, at any distance x, is given by

$$c_3 = \frac{\delta y}{w(L/2 - x)} = \frac{\delta x\left(4x^3 - 6Lx^2 + L^3\right)}{24EI\,(L/2 - x)} \qquad \ldots . \quad (A2.3)$$

If the beam is now considered as a series of panels (Fig 17) with

beam length $L = na$

panel length $\delta x = a$

distance $x = ma$

then equation A2.3 becomes

$$c_3 = \frac{a^4\left(4m^3 - 6nm^2 + n^3\right)}{24\,EI\,(n/2 - m)a} \qquad \ldots . \quad (A2.4)$$

In the context of a series of panels of depth b and cross sectional area of edge members A (Fig 18), then $I = Ab^2/2$ and equation A2.4 becomes

$$c_3 = \frac{a^3}{6EAb^2}(n^2 + 2nm - 2m^2) \qquad \text{.... (A2.5)}$$

The shear flexibility is a maximum when

$$d/dm\,(n^2 + 2nm - 2m^2) = 0 \quad \text{i.e.} \quad m = n/2$$

Then, the maximum shear flexibility

$$c_3\ \max = \frac{a^3}{6EAb^2} \times \frac{3n^2}{2} = \frac{n^2a^3}{4EAb^2} \qquad \text{.... (A2.6)}$$

If m = 0, then equation A2.5 gives the shear flexibility of the end panel

$$\text{i.e.} \quad c_3\ \text{end} = \frac{n^2a^3}{6EAb^2} \qquad \text{.... (A2.7)}$$

In particular, if the beam consists of only two panels, n = 2 in equation A2.7 and c_3 becomes $\frac{2a^3}{3EAb^2}$ which is the value used in Table 1.

Comparison with Equivalent Truss

Consider the beam shown in Fig 19, which is subdivided into 8 equal parts. Then, using equation A2.5, the shear flexibility at each of the points 1, 2, 3, 4 may be found by putting n = 8 and putting m = 0, 1, 2, 3 in turn.

Thus, at point 1, $$c_3 = \frac{32}{3} \times \frac{a^3}{EAb^2}$$

at point 2, $$c_3 = 13 \times \frac{a^3}{EAb^2} \qquad \text{.... (A2.8)}$$

at point 3, $$c_3 = \frac{44}{3} \times \frac{a^3}{EAb^2}$$

at point 4, $$c_3 = \frac{47}{3} \times \frac{a^3}{EAb^2}$$

Consider now the equivalent truss shown in Fig 19. If the areas of the vertical posts and diagonals are made infinite, the deflection is due to bending only, not shear, and it may be shown that the deflections at points 1, 2, 3, 4 are as follows:-

$$\delta_1 = \frac{42Wa^3}{EAb^2}, \quad \delta_2 = \frac{77Wa^3}{EAb^2}, \quad \delta_3 = \frac{100Wa^3}{EAb^2}, \quad \delta_4 = \frac{108Wa^3}{EAb^2}$$

The equivalent shear flexibilities are then as follows:-

$$\text{At point 1,} \quad c_3 = \frac{\delta_1}{{}^7/_2 W} = 12 \times \frac{a^3}{EAb^2}$$

$$\text{at point 2,} \quad c_3 = \frac{\delta_2 - \delta_1}{{}^5/_2 W} = 14 \times \frac{a^3}{EAb^2} \qquad \ldots. \quad (A2.9)$$

$$\text{at point 3,} \quad c_3 = \frac{\delta_3 - \delta_2}{{}^3/_2 W} = \frac{46}{3} \times \frac{a^3}{EAb^2}$$

$$\text{at point 4,} \quad c_4 = \frac{\delta_4 - \delta_3}{{}^W/_2} = 16 \times \frac{a^3}{EAb^2}$$

A comparison of the coefficients of equations A2.8 with those of A2.9 shows that the approximate beam theory is quite adequate to give the equivalent shear flexibility c_3 for most practical purposes.

APPENDIX 3 - ILLUSTRATIVE CALCULATION OF SHEAR DEFLECTION AND STRENGTH OF ROOF DECK (1) WITH PINJOINTED FRAMES AND (2) WITH RIGID FRAMES

Required

It is required to find the maximum shear deflection of the roof deck shown in Fig 20 and the load factor against shear failure. The horizontal design load is given. The supporting frames may be (1) pinjointed or (2) rigid jointed, as shown in Fig 21.

Data

The nomenclature is as given in Table 3.

Sheeting: $h = 63.5$mm, $d = 152$mm, $t = 0.9$mm, sheet width = 600mm, double span sheets

Edge members: 89mm x 89mm x 9.4mm equal angle, $A = 1592$ mm^2

Fasteners:
(i) Sheet/rafter fasteners - 3.7mm cartridge fired pins in every corrugation at the ends of the sheets and at sheet laps, and in alternate corrugations at intermediate rafters
(ii) Seam fasteners - 4.8mm Monel metal pop rivets at 400mm centres
(iii) Edge beam fasteners - 3.7mm fired pins at 400mm centres

Loading: Wind pressure p on side of building = 0.8 kN/m^2 so line load on roof deck = $^5/_2$ x 0.8 = 2.0 kN/m.

Derived Data

From reference 5, for the above sheeting, the K values are as follows:-

Fastened in every corrugation $K = 1.06$

Fastened in alternate corrugations $K = 29.9$

Hence take K as the mean of these, i.e. $K = 15.48$

For the fasteners, from reference 5, $s = 0.12$ mm/kN, $s_s = 0.35$ mm/kN, $s_{sc} = 0.12$ mm/kN. Also, $n_s = 9+1$, $n_{sc} = 10$, $n_{sh} = 20$, $p = 152$mm.

Note from para 3.9 that

$$\frac{\text{number of fasteners required on end rafter}}{\text{number of fasteners required on intermediate rafter}} = \frac{n}{2} = 4$$

Since this ratio is, in fact 2, the design criterion will be the fasteners on the end rafter.

Calculated Shear Flexibility

From the above values, using Table 1, it may be shown that the component flexibilities in the direction of the corrugations are: $c_{1.1} = 1.47$ mm/kN, $c_{1.2} = 0.074$ mm/kN, $c_{2.1} = 0.027$ mm/kN, $c_{2.2} = 0.056$ mm/kN, $c_{2.3} = 0.024$ mm/kN.

Summing these, the shear flexibility as shown in Fig 22(a) is 1.65 mm/kN.

It may be easily shown (reference 5) that the shear flexibility in the perpendicular direction, Fig 22(b), is given by $1.65 \times \left(\frac{b}{a}\right)^2 = 1.65 \times \left(\frac{4000}{12000}\right)^2 = 0.183$ mm/kN. In addition the value of c_3 is 0.014 mm/kN, so that the total shear flexibility in the perpendicular direction is given by $c = 0.197$ mm/kN.

Maximum Shear Deflection - Case (1)

For a design line load of 2.0 kN/m, the shear forces in the panels are as follows (see Fig 20):-

Panel 1-2, 32 kN; panel 2-3, 24 kN; panel 3-4, 16 kN; panel 4-5, 8 kN.

The maximum shear deflection, at frame 5, is then

$$\Delta_{max} = (32 + 24 + 16 + 8) \times 0.197 = \underline{15.8 \text{ mm}}$$

Shear Strength - Case (1)

Taking the tearing strength of the cartridge fired pin and the Monel metal pop rivet to be 6 and 2.5 kN per mm thickness of sheet respectively (ref 5), then

Tearing strength of fired pin = 6 x 0.9 = 5.4 kN

Tearing strength of pop rivet = 2.5 x 0.9 = 2.25 kN

(i) Strength of sheet/rafter fasteners in direction of load shown in Fig 22(b)

$$= 5.4 \times \frac{12000}{152} = 426 \text{ kN.}$$

(ii) Strength of seam fasteners in direction of load shown in Fig 22(a)

= 9 x 2.25 + 1 x 5.4 = 25.7 kN, so strength in direction of load in Fig 22(b)

$$= 25.7 \times \frac{12000}{4000} = 77.1 \text{ kN}$$

(iii) Strength of sheet/edge beam fasteners in direction of load shown in Fig 22(a)

= 10 x 5.4 = 54 kN, so strength in direction of load in Fig 22(b)

$$= 54 \times \frac{12000}{4000} = 162 \text{ kN}$$

Hence, the ultimate sehar strength is Q_{ult} = 77.1 kN, and the load factor against failure in the end panel is given by:

$$\text{Load factor} = \frac{77.1}{32} = \underline{2.4}$$

Maximum Shear Deflection - Case (2)

(a) For the portal frame shown in Fig 21(b), the horizontal load per frame is 8 kN, and the calculated sway deflection is 4.74 mm. Hence the frame flexibility $K = \frac{4.74}{8}$ = 0.59 mm/kN.

In the sheeted frames, the horizontal load is divided between the sheeting and the frames according to the number of frames and the "relative flexibility"

$$r = {}^{c}/k = \frac{0.197}{0.59} = 0.33.$$

For this value of r and for the 9 frames shown in Fig 20, reference 5 gives the following reduction factors to be applied to the bare frame sway moments and deflections:-

Frame 2, 0.42; frame 3, 0.65; frame 4, 0.73; frame 5, 0.80. Thus the maximum sway deflection at frame 5 is given by

$$\Delta_{max} = 0.80 \times 4.74 = \underline{3.8 \text{ mm}}$$

(b) An alternative method is to recognize that the forces acting on the sheeting at each frame may be obtained from the above reduction factors as follows:

$$F_2 = (1 - 0.42) \times 8 = 4.64 \text{ kN}$$

$$F_3 = (1 - 0.65) \times 8 = 2.80 \text{ kN}$$

$$F_4 = (1 - 0.73) \times 8 = 2.16 \text{ kN}$$

$$F_5 = (1 - 0.80) \times 8 = 1.60 \text{ kN}$$

These forces on the sheeting give rise to the panel shear forces shown in Fig 23 i.e. 10.40 kN, 5.76 kN, 2.96 kN and 0.80 kN respectively. The maximum shear deflection is thus

$$\Delta_{max} = (10.40 + 5.76 + 2.96 + 0.80) \times 0.197 = \underline{3.8 \text{ mm}}$$

Shear Strength - Case (2)

The ultimate shear strength, as before, is $Q_{ult} = 77.1$ kN, so the load factor against failure in the end panel is given by:

$$\text{Load factor} = \frac{77.1}{10.40} = \underline{7.4}$$

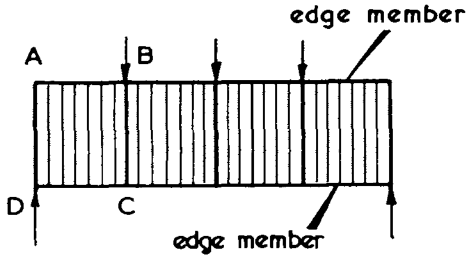


Fig 1. Concept of diaphragm plate girder

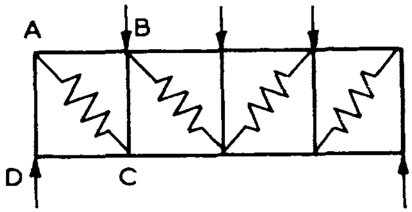


Fig 2. Equivalent springs in panels

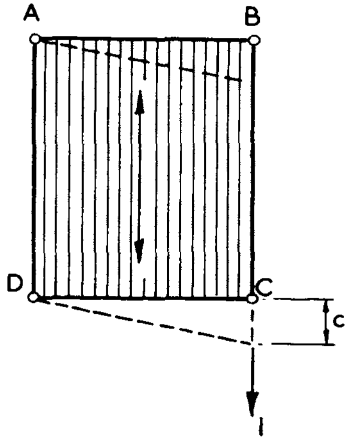


Fig 3. Definition of shear flexibility

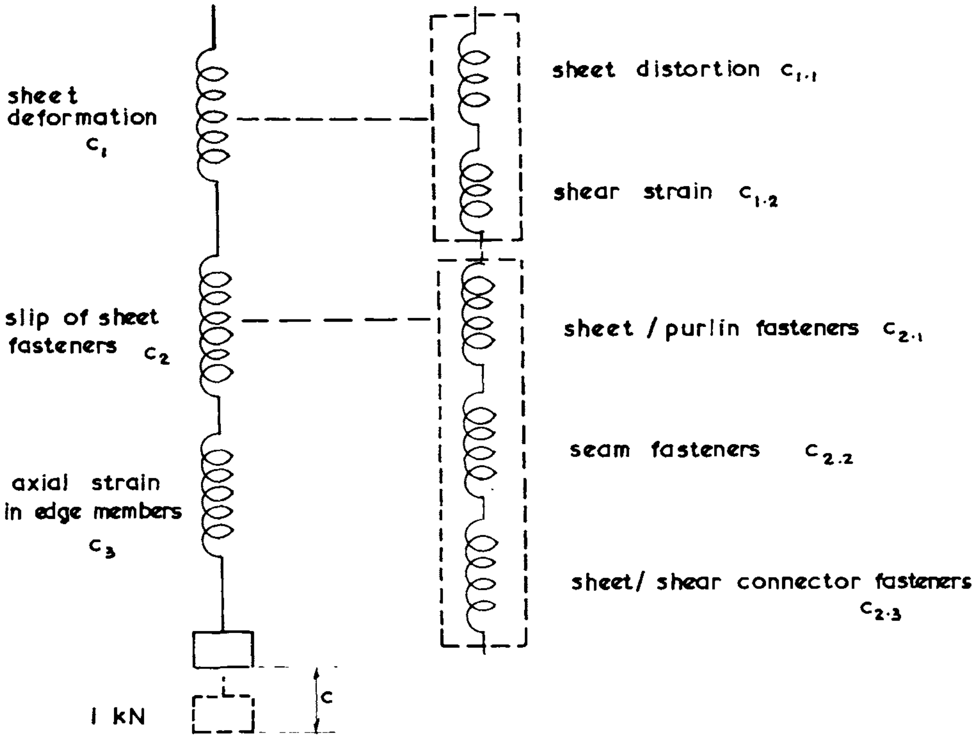


Fig 4. Spring analogy for shear flexibility

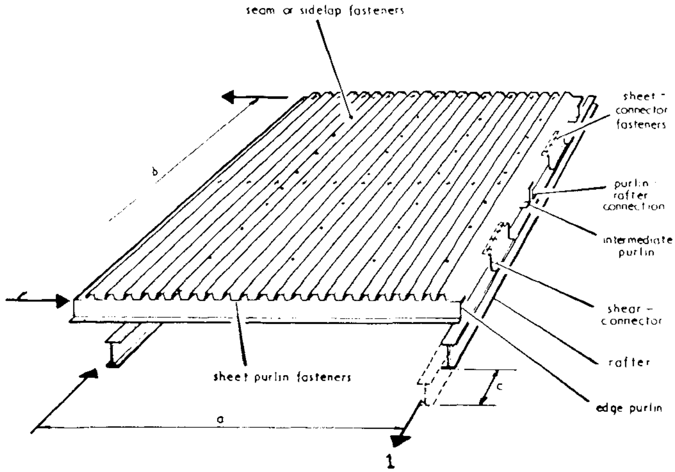
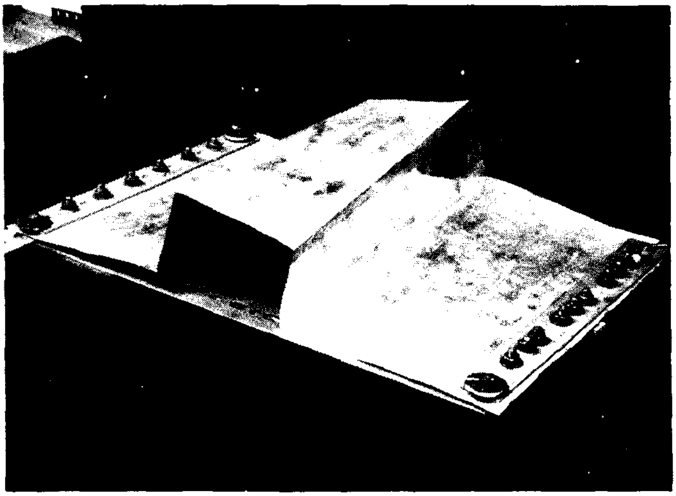
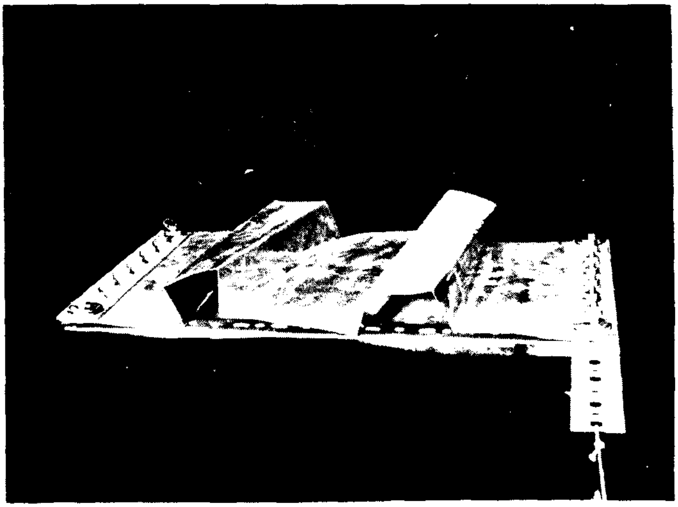


Fig 5. Arrangement of typical shear panel



(a) single corrugation



(b) double corrugation

Fig 6. Shear distortion of a corrugation

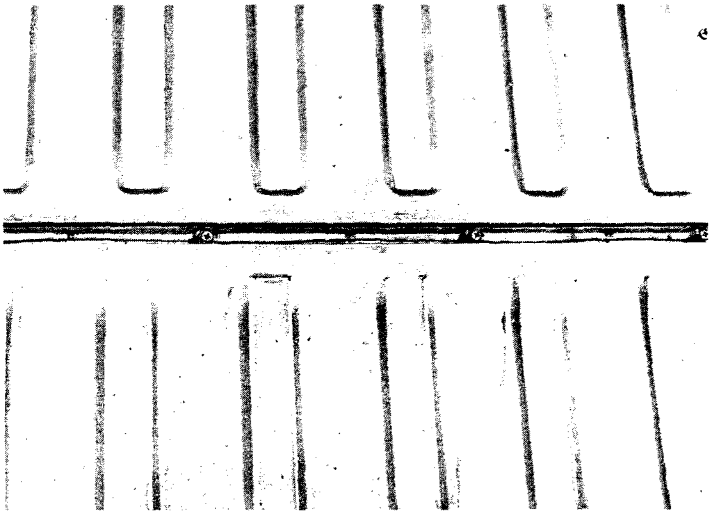


Fig 7. Slip in sheet/purlin fasteners

Fig 8. Slip in seam fasteners

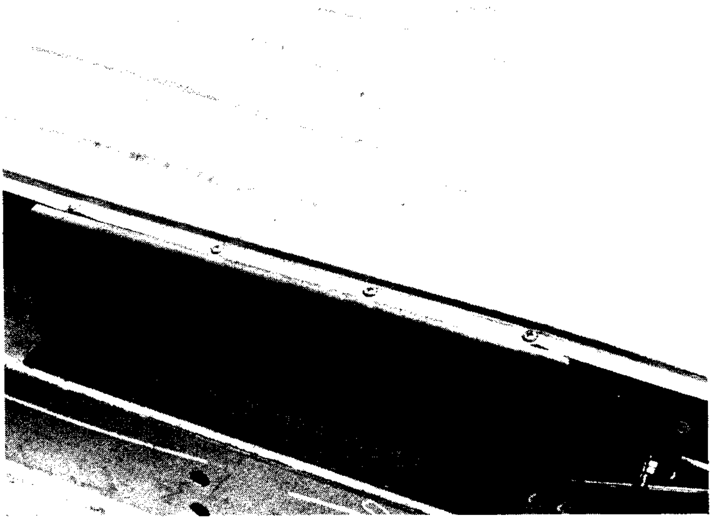


Fig 9. Slip in sheet/shear connector fasteners

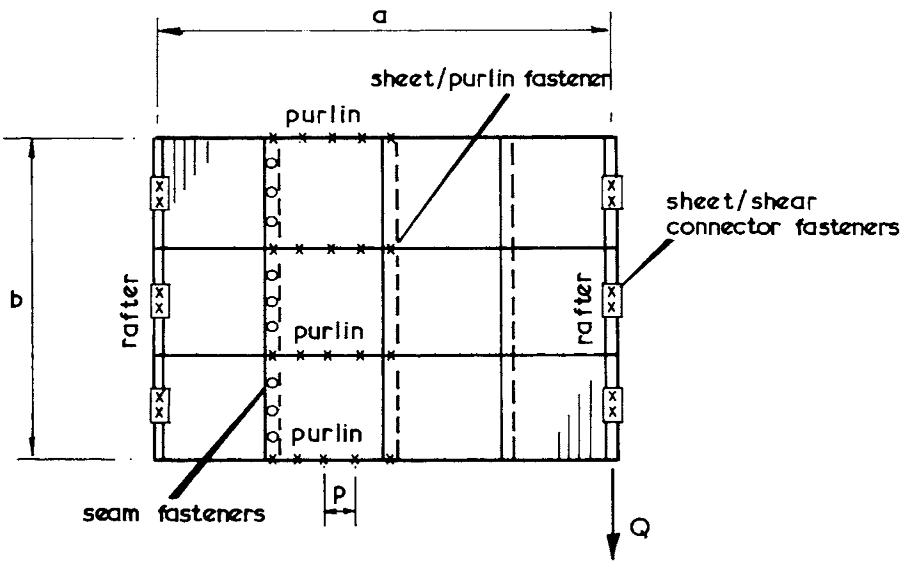


Fig 10. Typical panel arrangement with fasteners

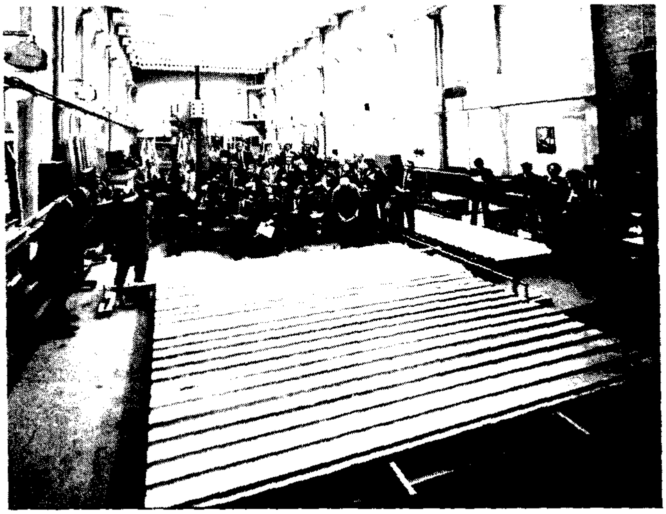


Fig 11. Demonstration test on shear panel

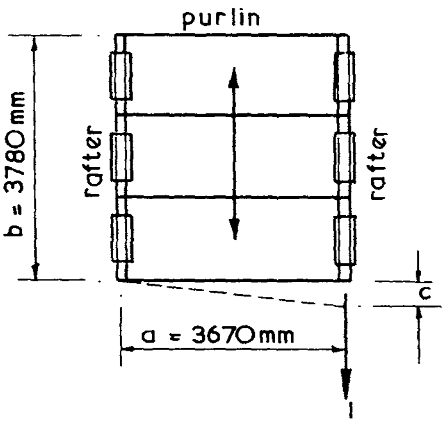


Fig 12. Dimensions of shear panel

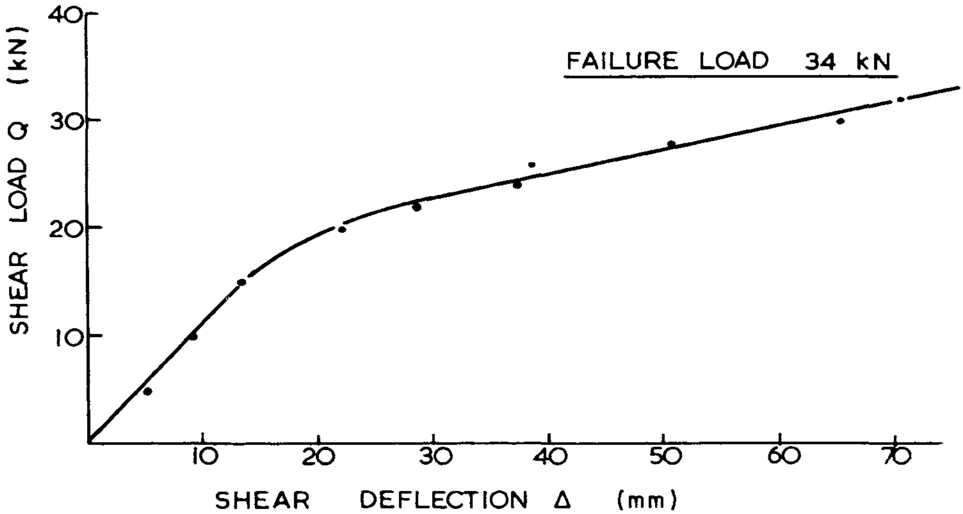


Fig 13. Observed load/deflection curve for shear panel

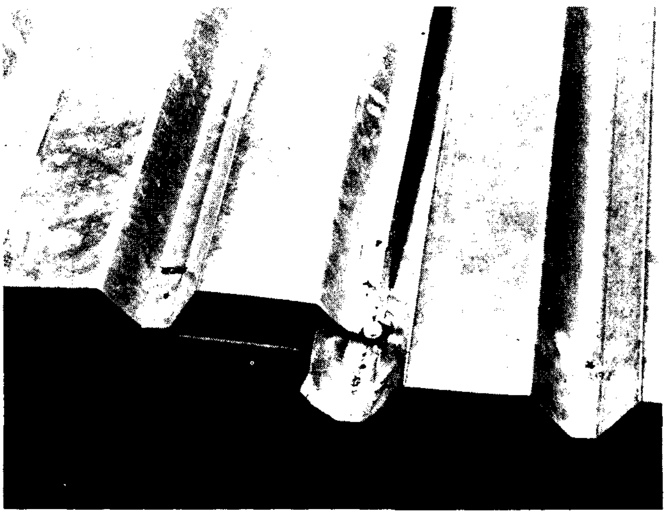


Fig 14. Demonstration test: failure of seam fasteners

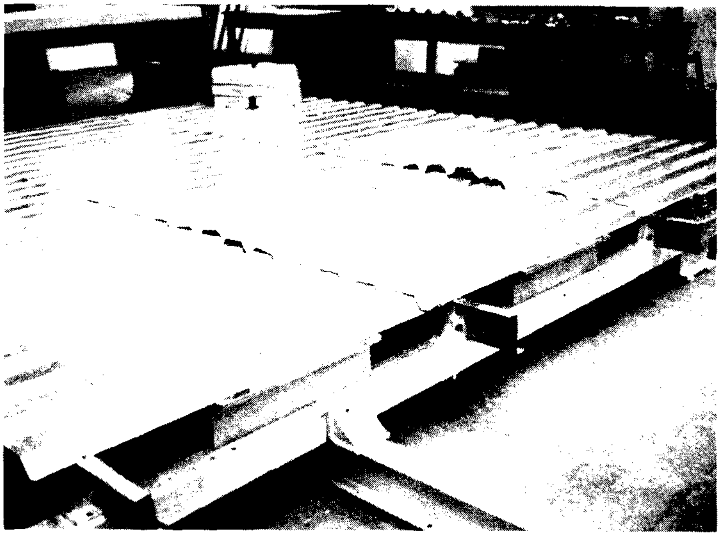


Fig 15. Demonstration test: three separate sheet lengths

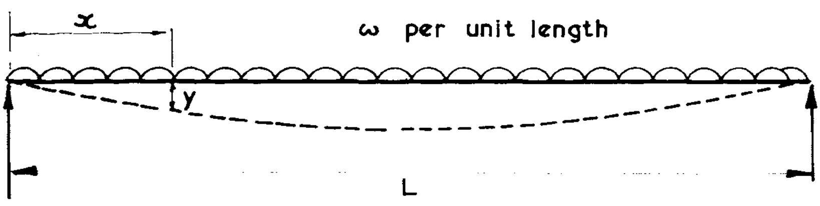


Fig 16. Bending deflection of beam

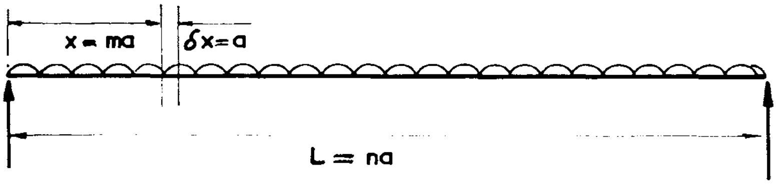
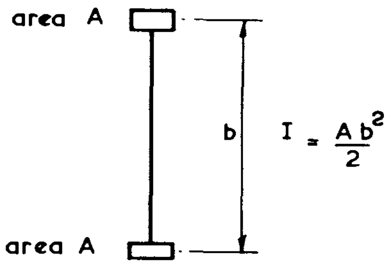


Fig 17. Beam considered as a series of panels

Fig 18. Second moment of area of beam



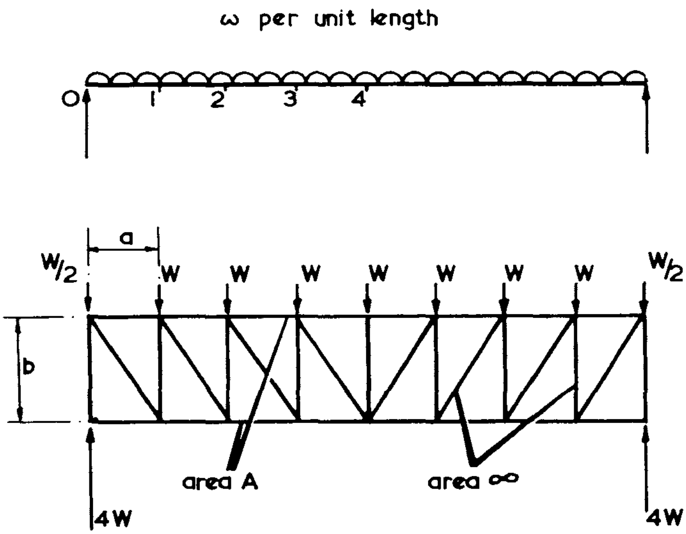


Fig. 19 Beam and equivalent truss

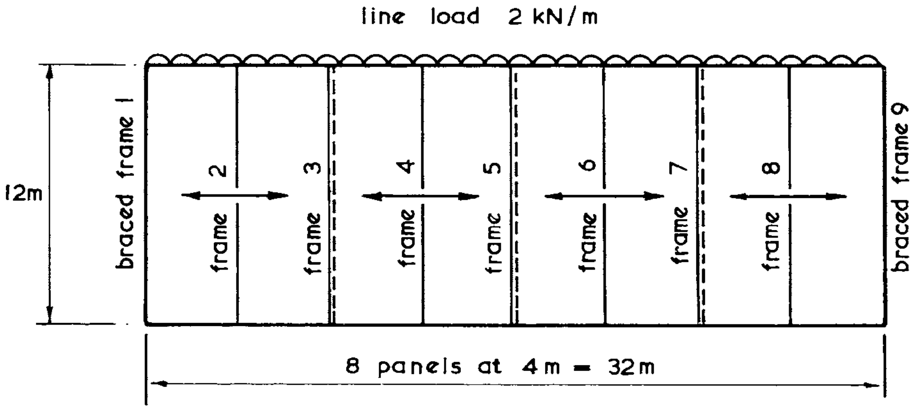


Fig 20. Plan of roof deck

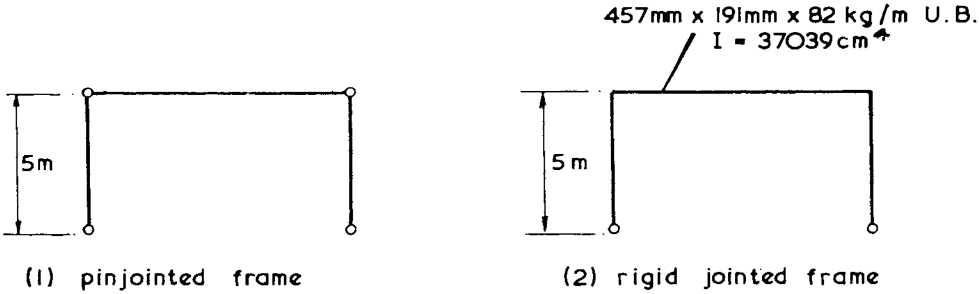


Fig 21. Details of supporting frames

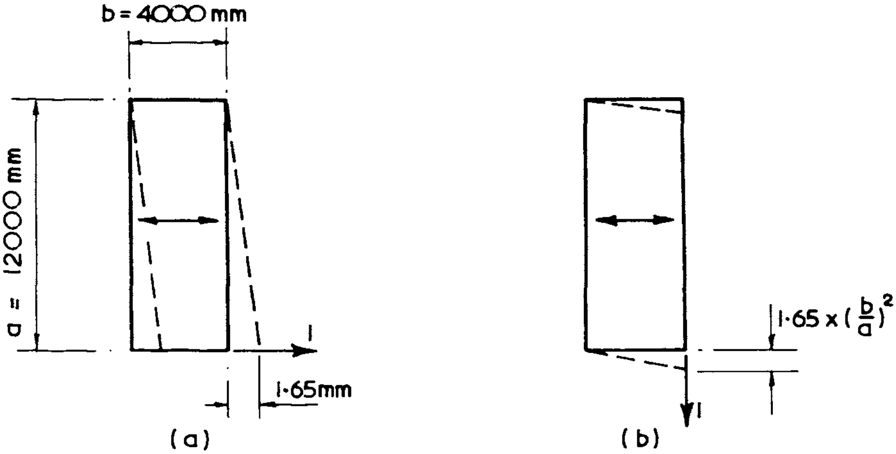


Fig 22. Shear flexibility in a perpendicular direction

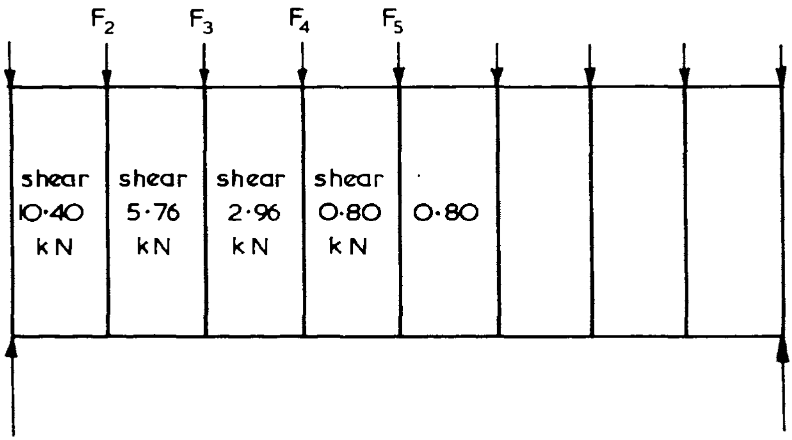


Fig 23. Shear forces in sheeting in case (2)